Modeling of small-diameter rotary drilling tests on marbles

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Abstract

In this paper, continuum mechanics and discrete modeling are applied to investigate numerically rotary drill cutting experimental results on four marbles. Rock-cutting tests were performed by a new portable rotary microdrilling tool currently employed in practice for the quasi-non-destructive characterization of strength properties of rocks. The objectives of this research work are twofold, namely: (a) to gain insight in the cutting mechanism of cohesive-frictional rocks, and (b) to examine the comparability of numerical models predictions with experimental results by solving the forward problem. In the first type of model, a plane strain continuum calculation is done with a non-hardening, elastic–plastic, linear Mohr–Coulomb model with non-associative flow rule. In the second type of numerical model, discrete element calculations are done on a simulated plane strain sample of 540 discs. In both models, estimations are made on components of force applied to the cutting face of the bit and are compared with measurements taken with the data-acquisition system of the portable microdrilling tool during specially designed tests on marbles. It is found that the predictions of the continuum model are in full accordance with measured forces during drilling. It is also shown that the cohesion and internal friction angle are the most important parameters affecting the rock drilling resistance, as is depicted by the limit analysis theory of plasticity. Moreover, the calibration of the discrete element model on the experimental data permits the approximate estimation of the mode-I fracture toughness for each type of marble.

Keywords: Cutting; Rotary drilling; Continuum model; Discrete element model; Marble

1. Introduction

In order to optimize restoration strategies for the protection of natural building stones in historical monuments or to secure reliable numerical model estimations of the mechanical behavior and collapse resistance of historical structures (i.e. arches, domes, vaults, masonry walls, etc.), it is necessary to know their mechanical properties, that is to say, deformability, uniaxial compressive and tensile strengths and fracture toughness among others, state of damage and effectiveness of possible consolidation treatments for restoration purposes (for example, depth of consolidant into the stone, enhancement of stone stiffness and strength, durability of consolidants, etc.). The quantification of the stone deformability, elasticity and strength, in situ, apart from numerical stability analysis, is also needed for the study of the decay process, and for better prediction and prevention of environmental damage to our cultural heritage. Further, it is also mentioned that the above rock properties are basic design parameters for many other rock-engineering applications (e.g. stability of excavations in rocks, cuttability of rocks by roadheaders and TBM’s among many others).

The methods employed so far for the determination of the compressive strength and the internal friction angle of rocks are based on loading a number of regular cylindrical-shaped specimens of rock in triaxial compression by employing special triaxial cells according to ISRM ‘Suggested Methods’[1]. When using regular-shaped specimens, such as diamond drill cores, it has been found experimentally that in many instances the results obtained are too scattered to be reliable, mainly because rocks are non-homogeneous and anisotropic. Further, the main disadvantages of the triaxial testing of rocks are two, namely:

1. It is destructive: a sample is taken from the structure—if it is permitted—it is cut and then on it rock mechanical tests are performed. On the other hand, if we want to use the structure thereafter it is mandatory that the
knowledge of the mechanical properties and damage should result from non-destructive tests.

2. The presence of any discontinuity in the specimen causes appreciable scatter to the results. It is therefore necessary in most cases to treat the results statistically, which necessitates a large amount of time and cost consuming experiments, in order to obtain the 'mean' cohesion and friction angle of the rock and their standard deviations.

Despite the obvious advantages of the use of small size drilling (e.g. drilling on small fragments and cores of few centimeters size) to assess quickly and in a quasi-non-destructive manner these stone properties, the method has not yet become a standard investigation tool in the rock mechanics community and in rock-engineering practice in general. The major obstacle pertaining to the application of the rotary drilling for the characterization of rocks, lies on the difficulty of back-analysis of drilling data in order to estimate their strength properties. This is due to the fact that the drilling measurements of stones are affected by many parameters, such as size, geometry and type of material of drill bit, friction between bit and stone, rotational and penetration speeds, heterogeneity of stone texture and physico-chemical setup, to name a few [2]. Nevertheless, the portable drilling force and torque measurement system (DFTMS) that it was developed in the frame of an EU funded project [3–6] represents a new cost and time-effective potential method for rock and stone mechanical properties determination if the influence of the various test parameters are understood and controlled.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
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<tbody>
<tr>
<td>$a$</td>
<td>bit radius</td>
</tr>
<tr>
<td>$c$</td>
<td>peak cohesion of rock</td>
</tr>
<tr>
<td>$d$</td>
<td>drilling depth reached by the bit</td>
</tr>
<tr>
<td>$E$</td>
<td>tangent deformation modulus of rock</td>
</tr>
<tr>
<td>$E'$</td>
<td>plane strain tangent deformation modulus of rock</td>
</tr>
<tr>
<td>$F_n$</td>
<td>total vertical force exerted on the cutter</td>
</tr>
<tr>
<td>$F_s$</td>
<td>total horizontal force exerted on the cutter</td>
</tr>
<tr>
<td>$K_{IC}$</td>
<td>mode-I fracture toughness of rock</td>
</tr>
<tr>
<td>$K_{pc}$</td>
<td>passive rock pressure coefficient</td>
</tr>
<tr>
<td>$K_n$</td>
<td>Ball–ball contact normal stiffness</td>
</tr>
<tr>
<td>$K_r$</td>
<td>Ball–ball contact shear stiffness</td>
</tr>
<tr>
<td>$n$</td>
<td>porosity of the rock</td>
</tr>
<tr>
<td>$P_{tot}$</td>
<td>passive pressure acting on the rigid edge of cutting tool that is in contact with the rock</td>
</tr>
<tr>
<td>$R$</td>
<td>distance of the point along the bottom cutting edge from the center</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>Mean ball radius</td>
</tr>
<tr>
<td>$T$</td>
<td>torque</td>
</tr>
<tr>
<td>$T_n$</td>
<td>contact-bond normal strength</td>
</tr>
<tr>
<td>$T_s$</td>
<td>contact-bond shear strength</td>
</tr>
<tr>
<td>UCS</td>
<td>unconfined compressive strength of rock</td>
</tr>
<tr>
<td>$v_p$</td>
<td>P-wave velocity</td>
</tr>
<tr>
<td>$u$</td>
<td>horizontal distance traveled by the cutter</td>
</tr>
<tr>
<td>$v$</td>
<td>cutter velocity</td>
</tr>
<tr>
<td>$W$</td>
<td>weight-on-bit</td>
</tr>
<tr>
<td>$\beta$</td>
<td>friction angle at the cutter–rock interface</td>
</tr>
<tr>
<td>$\delta$</td>
<td>cutting depth per revolution</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>ratio of normal to horizontal force</td>
</tr>
<tr>
<td>$\theta$</td>
<td>rake angle of cutter</td>
</tr>
<tr>
<td>$\mu$</td>
<td>friction coefficient of ball–ball contact</td>
</tr>
<tr>
<td>$\nu$</td>
<td>tangent Poisson’s ratio</td>
</tr>
<tr>
<td>$\nu'$</td>
<td>plane strain tangent Poisson’s ratio</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>major principal stress</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>minor principal stress</td>
</tr>
<tr>
<td>$\phi$</td>
<td>peak friction angle of rock</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>clearance angle of the drill bit</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>dilation angle of rock</td>
</tr>
<tr>
<td>$\omega$</td>
<td>drill bit rotational speed</td>
</tr>
</tbody>
</table>

2. Standard physico-mechanical properties of marbles and microdrilling test results

For the derivation of the standard rock mechanical properties of the rocks we employ a database of uniaxial and triaxial compression experiments on axisymmetric limestones and sandstones with uniaxial compressive strengths ranging from 1 to 110 MPa were carried out within the framework of an EU funded Project DIAS [6]. These experiments were performed by using a new portable microdrilling device that was first designed and constructed for the in situ evaluation of the hardness of stones (that is defined as the axial load on the cutter divided by the cross-sectional area of the cut for constant penetration rate and rotational speed). Subsequently, in a second phase of the tool development, the measurement of torque during drilling was also provided by the manufacturer (SINT Technology, Italy). This paper deals first with the numerical simulation of drill cutting experiments on a series of pure calcitic marbles performed by this tool. The main objective of this study is to establish whether or not results obtained in laboratory drilling experiments on marbles can be duplicated in numerical simulations. The rock drilling process is investigated using both the continuum and micromechanical approaches, by employing the two-dimensional dynamic explicit finite differences numerical code FLAC 2D [7], and the discrete (or distinct) element code PFC 2D [8], respectively. It is demonstrated that continuum model numerical results can be reproduced by an analytical model based on limit analysis theory of plasticity [9].
specimens with a diameter of 5 cm and a height of 10 cm, as well as direct tension and Brazilian tests on NX specimens according to ISRM suggested methods for these tests [1]. The main test results of each marble type are given in Table 1 [6,10] which lists the petrographical description, mean grain diameter \(D\), total porosity \(n\), bulk density \(\rho\), quartz content (SiO₂), type of binder material, mean tangent deformation modulus and lateral strain factor (or Poisson’s ratio) below 70\% of peak strength denoted by the symbols \(E\), \(v\), respectively, tensile strength TS, uniaxial compressive strength UCS, peak cohesion \(c\) and peak friction angle \(\phi\) (i.e. the failure criterion of marbles is approximated with a linear Mohr–Coulomb model). The marbles are contractant up to or near-peak axial stress and show a rather ductile post-peak behavior even at low or zero confining stress in compression. All types of marbles display a constant tangent deformability modulus \(E\) in a region \(0 < \sigma_a < \sigma_r\) (i.e. until the threshold stress \(\sigma_r\)), then decreases proportionally with axial stress \(\sigma_a\), and eventually becomes negative beyond the peak axial stress UCS. The tangent Poisson’s ratio increases with increasing axial stress for all types of tested marbles. As it may be seen from the same table, all types of marble are characterized by relatively high internal friction angles in the range 35° ≤ \(\phi\) < 40°. Also, in Fig. 1 the triaxial compression test results on these types of marbles are illustrated in the principal stress space.

The microdrilling tests were performed with the DFTMS, that is a new portable device which provides continuous measurements of the axial force or weight-on-bit (\(W\)) and torque (\(T\)) exerted on the cutter during the rotational penetration of the drill bit in the stone [5]. The mechanical part of DFTMS (Fig. 2a) consists of the tripod support, the stepping motor that is used for drill feed, the DC motor for rotation of the drill, the structure that is used to hold the specimen or to attach the wall, two load cells, the tightening chuck and two hardware limit stops. The drill bits that are used with this instrument have a small diameter in the range of 2–5 mm in order either to cause the minimum possible destruction of a structural element of rock or to perform a test on a small specimen (regular or irregular with a size of few cm’s). The drillhole depth may vary between 0.5 and 15 cm. During the drilling test the operating conditions are kept constant by pre-specifying the rotational speed \((\omega)\) and the penetration rate \((u)\) of the drill bit. The position of the bit referred to the surface of the stone is always known because it is controlled directly by the software through the dedicated electronics of the tool. The rotation speed of the drill can be set by the user from 0 to 1200 rpm at 100 rpm intervals and the penetration rate in the range of 0–40 mm/min at 5 mm/min intervals and they are both controlled by the electronic device in order to have a constant speed during the entire drilling procedure. The displacement of the drill is controlled with a stepper motor to give constant speed of movement with resolution of 0.0025 mm/step. The system measures continuously with a sampling frequency of

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Origin</th>
<th>Petrographical classification</th>
<th>Mean grain size (D) [(\mu)m]</th>
<th>Total porosity (n) [%]</th>
<th>Bulk density (\rho) [g/cm³]</th>
<th>Quartz content (SiO₂) [%]</th>
<th>Type of binder material</th>
<th>Mean tangent deformation modulus (E) [GPa]</th>
<th>Peak cohesion (c) [MPa]</th>
<th>Peak friction angle (\phi) [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gioia Marble</td>
<td>Italy, Carrara</td>
<td>Calcitic marble (homoblastic)</td>
<td>150–250</td>
<td>1.5 ± 0.2</td>
<td>2.67 ± 0.1</td>
<td>traces</td>
<td>None</td>
<td>65</td>
<td>0.23</td>
<td>7.5</td>
</tr>
<tr>
<td>Cervaiole Marble</td>
<td>Italy, Carrara</td>
<td>Calcitic marble (xenoblastic)</td>
<td>300</td>
<td>1.3 ± 0.3</td>
<td>2.71 ± 0.2</td>
<td>traces</td>
<td>None</td>
<td>60</td>
<td>0.27</td>
<td>9.4</td>
</tr>
<tr>
<td>Lorano Marble</td>
<td>Italy, Carrara</td>
<td>Calcitic marble (xenoblastic)</td>
<td>200–250</td>
<td>1.1 ± 0.1</td>
<td>2.67 ± 0.1</td>
<td>traces</td>
<td>None</td>
<td>60</td>
<td>0.29</td>
<td>5.6</td>
</tr>
<tr>
<td>Dionysos marble</td>
<td>Greece, Athens</td>
<td>Calcitic marble (homoblastic)</td>
<td>300–400</td>
<td>1.6 ± 0.2</td>
<td>2.66 ± 0.1</td>
<td>traces</td>
<td>None</td>
<td>80</td>
<td>0.33</td>
<td>8.2</td>
</tr>
</tbody>
</table>
100 Hz: (a) the axial force (100 N maximum allowed load),
(b) the torque (500 Nmm maximum allowed torque) via the
two load cells, respectively, and the actual drill bit position.
All logged data and test data settings are transmitted
directly to a PC through a standard serial communication
port by employing LabView software. The output of the
system pertains to numerical and graphical reports for
subsequent numerical processing.

The drill bits used for the experiments are coming from
normal masonry practice and are made by soldering a tip
of polycrystalline diamond (PCD) as it is illustrated in
Figs. 2b,c. As it may seen from the drawing of Fig. 2c the
clearance angle of the drill bit is 10°, its rake angle is 30°
and its diameter is 5 mm.

The rotary drag bit bores into the rock along a helical
path under the combined action of an axial thrust, \( W \), and
a rotary torque, \( T \), planning-off the rock ahead of it.
Several drilling tests were performed on the marbles in
order to measure the drilling thrust and torque reactions on
the drill bit with respect to the cutting depth \( \delta \) per
revolution. According to basic principles of differential
geometry the cutting depth per revolution for a circular
helix is defined as follows:

\[
\delta = \frac{2\pi n}{\omega}.
\]  

(1)

The experimental data of this study for cutting depths \( \delta \)
in the range 0.03–0.55 mm/rev in marble disc specimens of
54 mm diameter and 20 mm thickness have been initially
subjected to data smoothing. As an example, we refer here
to a particular drilling record of force and torque logs of
Dionysos that is displayed in Figs. 3a,b. As it may be seen
from these figures the axial force and torque signals
exhibit a ‘transient part’ and a ‘steady-state part’.
In order to obtain the steady-state values of \( W \) and \( T \), these
primary data have been smoothed by passing the best-fit
exponential curve

\[
\begin{align*}
\{ W \} & = \{ \tilde{W} \} + \left\{ \begin{array}{c}
 b_1 \\
 b_2 
\end{array} \right\} \exp(-d),
\end{align*}
\]  

(2)

where \( \tilde{W} \), \( \tilde{T} \), \( b_1 \), \( b_2 \) are best-fit parameters, and \( d \) denotes
the recorded current drilling depth. The steady-state values of
weight-on-bit and torque are recovered for \( d \to \infty \), and
are denoted in Eq. (2) by the symbols \( \tilde{W} \), \( \tilde{T} \), respectively.

The resultant vertical and horizontal forces acting on the
face of the cutter in contact with the rock are denoted with
the symbols \( F_n (= \tilde{W}) \) and \( F_s \). These cutting forces lie in the
same vertical plane where the cutting velocity vector also
lies. Due to symmetry no horizontal force orthogonal to
the direction of the cut is expected. According to basic
principles of mechanics of rotary drill cutting we may find that

\[
F_s = \frac{\tilde{T}}{(a/2)}. 
\]  

(3)

Plots of the vertical and horizontal resultant forces
acting on the face of the cutter versus the drilling
depth per revolution are shown in Figs. 4a and b,
respectively. Of particular interest are: (a) the value of
the slope of \( F_n \) versus \( F_s \) curve in a particular drilling
experiment, and (b) the dependence of this slope on the
interfacial friction angle of the cutter–rock interface,
indicated here with the symbol \( \zeta \), is constant, thus independent from \( \delta \) (i.e. there is no size
effect), and it is more or less the same for all types of
marbles (Fig. 4c).

With reference to the conceptual model of the
perfectly sharp cutter with a rake angle \( \theta \) tracing a
groove on a horizontal surface with velocity \( v \) (Fig. 5)
and the phenomenological relationship of Fig. 4c, the
increments of the vertical and horizontal forces, \( dF_n \), \( dF_s \)
respectively, applied to the tool should be related as follows:

\[
dF_n = \zeta dF_s. 
\]  

(4)

From this type of elaboration of experimental results,
the interfacial friction angle of the cutter–rock interface \( \beta \)
may be found by virtue of simple geometry as follows:

\[
\zeta = \tan(\theta + \beta),
\]  

(5)

According to Eq. (5) and the estimated slope of
\( dF_n/dF_s \approx 0.6 \) from Fig. 4c, it may be found that
\( \beta \approx 0^\circ \), thus indicating that almost perfectly smooth
contact conditions are prevailing during the cutting
experiments.

The non-zero intercept of the forces measured during the
drilling tests is due to the fact that a small flat region under
the tip exists (i.e. the tip is not a perfectly sharp wedge).
This flat region, that is of the order of 0.1 mm, is not negligible compared to depths of cut seen by the cutter during the experiments. As it will be seen later this effect is removed by subtracting the intercepts from both measured contact forces in order to get only the cutting components of these forces.
3. Continuum modeling of rock cutting

For the simulation of the above rock–cutting experiments, the 2D explicit finite differences numerical code FLAC2D (version 4.0) [7] is employed. First, the geometry of the cutting face, that is prescribed by the geometry of the tool and the operational drilling parameters, is properly depicted for the numerical model. In the model the cutting depth is taken to be constant, whereas in reality the bit follows a helical path and the penetration increases linearly with depth in one complete revolution of the bit; however, as it was mentioned above, the cutting depth per revolution \( \delta \) during drilling remains constant. In this approach, the effective cutting depth in the model is such that the cutting volume per revolution is equal to the respective cutting volume per revolution of the drill bit.

The discretization of the numerical model is dense in the region close to the inclined wall in order to capture the localization process. The cutting action (tool–rock interaction) is simulated here by prescribing constant horizontal translational velocity \( v \) to the right of all nodes belonging to the inclined wall (Fig. 6a), whereas the tractions are prescribed to be zero on the upper horizontal surfaces and roller restraints on vertical edges and bottom edge of the model. A relatively low velocity was prescribed to approximate quasi-static loading conditions, since

\[
\frac{v}{v_p} \ll 1,
\]

where \( v_p \) is the compressive wave velocity of the rock. The validity of this assumption may be checked by the values of \( v_p \)-velocities of the marbles presented in Table 1, the formula for cutting speed \( v = \omega r \) (\( r \) = distance of the point along the bottom cutting edge from the center) and the maximum rotational speed \( \omega = 1200 \text{ rpm} \) employed during the drilling tests. Also, plane strain conditions are justified by the fact that the cutting depth per revolution is much smaller than the drillhole radius, i.e.

\[
\frac{\delta}{a} \ll 1. \tag{7}
\]

The mechanical behavior of the rock is approximated by an elastic–perfectly plastic model with non-associative flow rule. For the constitutive law we employ Mohr–Coulomb plasticity, which consists of a linear composite yield function with tension cut-off that governs the onset of plastic behavior and a plastic potential function which describes the flow rule. Herein, compressive stresses are taken as negative. The dilation angle \( \psi \) is the ratio of the plastic volume change over the plastic shear strain and is positive for volume increase with increasing shearing strain. It is remarked here that this type of model does not take into account the indentation mechanism that is also operating during the initial tool–rock contact phase of drilling, but only the ‘passive rock pressure’ mechanism that is responsible for rock chipping (i.e. in a fashion similar to the Merchant mechanism [11–13]).

In all the simulations a null dilatancy angle is assumed, \( \psi = 0 \), i.e. in localization zones the material experiences only shear distortion and zero dilation. All the remaining physico-mechanical properties of marbles (i.e. \( \rho, TS, c, \phi, E \) and \( v \)) were taken from Table 1. The initial state of the model is shown in Fig. 6a. The distorted finite difference grid, as well as contours of maximum shear strain rate corresponding to successive amounts of sliding \( u \) along the horizontal plane are illustrated in Figs. 6b–f. The results of the numerical simulations elucidate also the failure kinematical mechanism of the continuum model. Initially a horizontal shear band is formed that starts from the lower corner of the contact region (Fig. 6b). At a subsequent stage a second shear band develops from the horizontal shear band extending up to the upper horizontal edge of the model (Fig. 6c). Since further deformation cannot be accommodated by these two shear bands, a new shear band starts to grow from the upper corner of the contact region (Fig. 6d), that finally meets the other two shear bands at their intersection point (Fig. 6e). The final fully developed kinematical failure mechanism is illustrated in Fig. 6f. Thus, it is observed that well-defined shear bands are seen to develop even for an non-softening material, without a weak defect or inhomogeneity being present at the start of the simulation. Also, according to the

![Graphs](image-url)
kinematical failure mechanism predicted by this model, the rock fails under a continuous plastic crushing process rather than in a discontinuous brittle chipping process.

In Fig. 7, an enlarged view of the stress distribution near the horizontal shear band is displayed. The perturbation in stress in the shear band conforms well with the conceptual picture of localization proposed by Cundall [14] which depicts principal stress rotation of $\pm 45^\circ$ relative to the band and a reduction in magnitude.

Several simulations have been carried out in order to study the influence of the cutting depth $\delta$ on the force–displacement behavior of the tool for each type of marble. For example, Fig. 8 displays the variation of the horizontal and vertical components of the thrust force acting on the cutter with the horizontal displacement $u$ for three cutting depths in Dionysos marble.

Subsequently, the mean values of horizontal and vertical cutter forces calculated for the various simulated cutting depths have been plotted in Figs. 9a–d. In the same figures the measured in the laboratory cutting forces by DFTMS are also displayed for comparison purposes. As it was mentioned in the last paragraph of Section 2 the experimental results were corrected by subtracting the constant terms found by interpolating these data by linear curves. Interestingly, model predictions are in very good agreement with the drill cutting measurements on all marbles. It may be also noticed that both measured and predicted cutting forces are proportional to the cutting depth, which justifies the use of the linear Mohr–Coulomb failure criterion.

4. Discrete element modeling of rock cutting

As it is displayed in Fig. 10, experimental evidence clearly indicates the granular microstructure of marbles.
Thus, one may argue that the previous continuum modeling approach is not adequate, since the granular structure of marble (i.e. grain size and shape, type of particle interlocking, etc.) may influence the cutting process. Further, one may also argue that the continuum model simulates a steady-state process and not a truly destructive process as really cutting of rocks is. Moreover, another argument could be raised, namely, that the continuum model captures only the onset of failure and not the complete cycle of the fracture process. For all these purposes rock-cutting simulations have been performed here by using the discrete element method. In this case, the granular rock is represented with an assembly of bonded, circular particles with program PFC\textsuperscript{2D} (Particle Flow Code of 2 Dimensions) [8]. The drill bit is modeled as a set of walls that move at a specified constant horizontal velocity and depth of cut on a horizontal surface of a granular rock while monitoring traction forces applied on the cutter and evolving damage in the rock. The process of rock ‘fracturing’ consists of progressive bond-breakage.

In order to set up a discrete element model three fundamental components of the model must be specified, namely:

1. The assembly of circular particles: It consists of the locations and size distribution of particles (simulated here as circular discs) (Fig. 11). A 2D assembly of rods packed together as shown in Fig. 11 is called Schneebeli material. The particles are randomly generated within a
rectangular frictionless frame and the radii of the particles are normally distributed according to a prescribed standard deviation from a mean radius $\bar{R}$.

2. The contact behavior and material properties of the assembly: They dictate the type of response the model will display upon disturbance (e.g. deformation response due to biaxial loading).

3. The boundary and initial conditions: These conditions prescribe the initial in-situ stress state of the model before any disturbance is introduced into the model.

The motion and the interaction of the rigid circular particles in PFC$^{2D}$ contact-bonded model are established by the following three sets of parameters $[8,15]$:

(a) Geometric and physical parameters:

- $\bar{R}\left[F \cdot L^{-1}\right] = \text{average ball radius,}$
- $\rho\left[F \cdot L^{-3}\right] = \text{density of particles,}$
- $n[-] = \text{porosity of the particle assembly.}$

(b) Mechanical parameters for the contacts between the particles:

- $K_n\left[F \cdot L^{-1}\right] = \text{ball–ball contact normal stiffness,}$
- $K_s\left[F \cdot L^{-1}\right] = \text{ball–ball contact shear stiffness,}$
- $T_n\left[F \cdot L^{-1}\right] = \text{contact-bond normal strength,}$
- $T_s\left[F \cdot L^{-1}\right] = \text{contact-bond shear strength,}$
- $\mu[-] = \text{ball–ball contact friction coefficient.}$
Loading condition parameters:
\[ L = \text{characteristic dimension of the model}, \]
\[ v = \text{applied wall velocity}, \]

wherein \([L, F, T]\) denote the fundamental dimensions of length, force and time, respectively.

The contacts between particles are described through the stiffness, slip, and bonding relations. Herein, a linear force–displacement constitutive law at each contact is assumed, namely
\[ F_n^i = K_n U_n^i, \]
\[ \Delta F_s^i = -K_s \Delta U_s^i, \]  
(8)

where \(F_n^i, F_s^i\) are the normal and shear components of the contact force; \(U_n^i, U_s^i\) are the displacement components along the normal and tangential directions of the contact, respectively. Further, a linear Coulomb-type failure criterion at the ball contacts has been adopted
\[ F_n^i = F_s^i = 0 \quad \text{if} \quad F_n^i \geq T_n \bar{R}, \]
\[ |F_s^i| = \mu |F_n^i| \quad \text{if} \quad F_s^i \geq T_s \bar{R}. \]  
(9)

Before the modeling of the rock-cutting process, the ‘synthetic granular material’ is calibrated by simulating standard laboratory tests, and adjusting micro-properties until a good quantitative match is obtained for the following rock mechanical properties:

1. Tangent deformation modulus.
2. Tangent Poisson’s ratio (or tangent lateral strain factor).
3. Uniaxial compressive strength.

4. Strength envelope expressed by the variation of the peak axial stress versus confining radial stress in an axisymmetric triaxial compression configuration.

The set of micromechanical properties for all the marbles were found by virtue of phenomenological relationships in dimensionless form
\[ \frac{E \bar{R}}{K_n} = f_1 \left( \frac{K_n}{K_s} \right), \]
\[ \nu = f_2 \left( \frac{K_n}{K_s} \right), \]
\[ \frac{c \bar{R}}{T_n} = f_3 \left( \frac{T_n}{K_n}, \frac{K_n}{T_n} \right), \]
\[ \phi = f_4 \left( \frac{T_n}{K_n}, \frac{K_n}{T_n} \right). \]  
(10)

The above relations were derived by carrying out a series of 2D uniaxial and triaxial compression discrete element simulations.

The discrete element code is inherently 2D, hence it may be interpreted as representing either a plane stress or plane strain condition. Herein, as it was explained above, plane strain conditions are assumed, therefore the 2D constants \(E\) and \(\nu\) can be related to \(E\) and \(\nu\) according to the relations
\[ E' = \frac{E}{1 - \nu^2}, \nu' = \frac{\nu}{1 - \nu}. \]  
(11)

For example, Table 2 displays the set of micromechanical parameters of Dionysos marble that give the best possible comparability between the discrete element model with the uniaxial and triaxial compression experimental tests on this marble. As it may be noticed from this table, the marbles are characterized by high \(T_s/T_n\) ratios in order to capture the large internal friction angles exhibited by them. Fig. 12a shows the distribution of the tensile and compression contact forces inside the specimen that is subjected to unconfined compression. Further, Figs. 12b–e display the comparison of the calibrated model predictions and test results pertaining to the mechanical behavior of Dionysos marble in unconfined compression. The determination of the deformability modulus \(E\) and the lateral strain factor \(\nu\) was made along the slope of the tangent lines drawn at the axial stress \(\sigma_a\) of the axial stress–axial strain curves and lateral strain–axial strain curves, respectively.

![Fig. 11. Initial condition of an assembly of 540 particles (circular discs) confined between walls for the simulation of a biaxial compression test and bonds (lines) between the balls.](image)

Table 2

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ball density (kg/m³)</td>
<td>(\rho)</td>
<td>2700</td>
</tr>
<tr>
<td>Porosity of the particle assembly</td>
<td>(N)</td>
<td>0.17</td>
</tr>
<tr>
<td>Ball stiffness ratio</td>
<td>(K_a/K_s)</td>
<td>2</td>
</tr>
<tr>
<td>Ball normal stiffness (GN/m)</td>
<td>(K_n)</td>
<td>250</td>
</tr>
<tr>
<td>Ball friction coefficient</td>
<td>(\mu)</td>
<td>0.50</td>
</tr>
<tr>
<td>Contact-bond normal strength (MN/m)</td>
<td>(T_n)</td>
<td>0.11</td>
</tr>
<tr>
<td>Contact-bond shear strength (MN/m)</td>
<td>(T_s)</td>
<td>0.9</td>
</tr>
</tbody>
</table>
i.e.

\[ E = \frac{\partial \sigma_a}{\partial \varepsilon_a}, \quad \nu = -\frac{\partial \sigma_a}{\partial \varepsilon_r}, \]

(12)

wherein \( \varepsilon_a \) denotes the axial strain and \( \varepsilon_r \) denotes the lateral (or radial) strain recorded during the uniaxial compression tests.

The calibrated discrete element models on the experimental uniaxial stress-strain curves of the remaining three

![UC: Dionysos marble](b)

![Axial strain [x1E+3]](c)

![Axial stress [MPa]](d)

![Tangent lateral strain factor](e)

![UC: Dionysos marble](f)
marble types are also illustrated in Appendix A. Furthermore, a series of triaxial compression tests on the same rectangular disc assembly (i.e. synthetic rock specimen) have been also simulated in order to verify that the estimated bond strength parameters (Table 2) result to good comparability with experimental data referring to triaxial compression tests. The good agreement of the models with experimental results for the four types of marble is illustrated in Fig. 13 where the peak axial stress in all tests is plotted against the confining stress.

Next, the calibrated granular models of each type of stone are subjected to rotary drill cutting simulations with various cutting depths with the geometry of the cutter resembling that of the drill bit of the microdrilling device (Fig. 2c). The cutting process is simulated by moving the cutter horizontally to the right (Fig. 14) at a relatively low velocity to approximate quasi-static loading conditions, and in plane strain conditions (as in the continuum model case). The cutter is represented by a single wall of two segments that form a rake angle of 30° and a clearance angle of 10°. The model contains 540 balls, it has a rectangular shape and the balls on the bottom and right-hand sides of the specimen are fixed. The contact between particles and the cutter obeys a frictional law, with a friction coefficient taken to be also μ = 0.5.

During these simulations the following parameters are monitored as a function of the relative cutter position s/R:

1. Exerted forces on the cutter, namely the total horizontal force $F_x$ and the total vertical force $F_y$ (as in the case of the continuum model).
2. Damage that is depicted by the intensity, location and orientation of the microcracks, and their possible coalescence into macrofractures.
3. Force distribution within the granular model, i.e. of compressive forces arising at particle–particle contacts when the bonds are broken and compressive, shear and tensile forces acting in the bonding material.

The simulations revealed the following features of the rock-cutting cycle:

- Build-up of forces as the bit contacts the granular rock, until microfractures are generated due to bond breaking between particles (Fig. 14). Slight falling-off of forces as minor fractures occur.
- Coalescence of microfractures into a macrofracture of a log spiral geometry extending from the bit tip to the rock surface (global maximum of the cutting cycle).
- Rapid release of stored energy in the bit and falling of cutting forces.
- Build-up of forces again until a major fracture occurs, and the cycle is repeated.

Typical numerical results pertaining to the variation of cutting forces with sliding distance of the cutter that correspond to a small and a large cutting depth, respectively, are displayed in Figs. 15 and 16. The local peak values in the force log indicate breaking of bonds between particles, while the higher peaks correspond to either the coalescence of microfractures or the formation of rock chips. As it may be noticed from Fig. 16 the forces exhibit a slight positive slope which is caused by the additional resistance exerted on the cutter due to increasing volume of the loose material.

The dependence of the mean values of the horizontal force over the distance traveled by the cutter on the relative cutting depth $\delta/R$, predicted by the distinct element model for all the marbles is plotted in log–log diagrams (Figs. 17–20), respectively. The mean phenomenological particle radius $R$ for each type of marble is found by best matching the slopes of the experimental and numerical simulations regression lines of the tangential force versus. relative cutting depth. The best estimations of the mean particle radius of each type of marble are displayed in Table 3. The concave upwards shape of the curves obtained from the distinct element simulations for small relative cutting depths $\delta/R$, is due to the fact that the present discrete element model cannot simulate particle crushing into smaller particles due to transgranular cracking. Particle crushing during drilling is demonstrated by comparing the original grain size cumulative distribution with the particle
size cumulative distribution of the marble (it is inferred that the original mean grain size of marble is reduced approximately 30 times by rotary drill cutting). Also, the nonlinear shape of the test data in the log–log diagram is due to the fact that they display a non-zero intercept due to some flatness of the bit. In the same figures the predictions of the FLAC models are displayed after adding to these the constant intercept term found from experiments. These diagrams demonstrate the good agreement of the three sets of data, namely test data, FLAC elastoplastic modeling predictions and PFC discrete element predictions.

The inclination $\zeta$ of the resultant cutting force vector can be also derived from the slope of the $F_n - F_s$ linear curve that is derived from the discrete element simulation tests (Fig. 21). This value of inclination is in close agreement to that predicted by the continuum element model and to that obtained from the experiments (e.g. Fig. 4c).

It can be shown that the average initial crack half-length in a circular particle assembly is approximately equal to the average radius $\bar{R}$ of the circular particles. Hence, according to first principles of linear elastic fracture mechanics, the plane-strain mode-I fracture toughness $K_{IC}$ for this particulate material can be expressed as

$$ K_{IC} = TS' \sqrt{\pi \bar{R}} $$

(13)

where $TS'$ is the tensile strength estimated from simulated Brazilian tests by using the calibrated micromechanical models for each type of marble. The values of the fracture toughness of each marble estimated by the above formula are displayed below in Table 3. Along with these values, the estimated mode-I fracture toughness values of two marbles with standard fracture mechanics testing are also displayed for comparison purposes. The only available values of mode-I fracture toughness are those of Dionysos marble experimentally determined from single edge notch
specimens in three-point bending independently by two laboratories, i.e. \(K_{IC} = 2.0 \text{ MPa} \sqrt{m}\) [16] and \(K_{IC} = 1.9 \pm 0.5 \text{ MPa} \sqrt{m}\) [17], and of Lorano marble, namely \(K_{IC} = 0.7 \pm 0.3 \text{ MPa} \sqrt{m}\), that was determined from pre-cracked Brazilian disc tests [18]. These values are in good agreement with the corresponding values found indirectly from the distinct element simulations (Table 3).

### 5. Analytical solution

The upper bounds of cutting forces \(F_p, F_s\) for a given cutting depth per revolution \(\delta\) may be found by recourse to the upper bound theorem of limit analysis [9]. In such a type of analysis the first step is the construction of a realistic kinematical failure mechanism. Herein, the kinematical failure mechanism ensued from the numerical algorithm (Fig. 6) is approximated with two triangles as it is displayed in Fig. 22.

By ignoring the weight of the rock, the expression for passive failure load \(P_{tot}\) corresponding to the above kinematical failure mechanism occurring in a rock obeying the associated linear Mohr–Coulomb failure criterion (i.e. for \(\psi = \phi\)) is given by the following equation [9]:

\[
P_{tot} = c\delta K_{pc}(\phi, \beta, \theta, P, \Omega)
\]  

(14)

where \(P_{tot}\) is the passive pressure acting on the rigid edge of cutting tool that is in contact with the rock. For \(\beta \cong 0\), i.e. for perfectly smooth contact, it can be shown that the passive rock pressure coefficient is expressed as a function of internal friction angle, rake angle and the two angles of the two-triangle mechanism \(P, \Omega\) (e.g. Fig. 23) with the following formula:

\[
k_{pc}(\phi, \theta, P, \Omega) = \frac{1}{\cos(\theta)} \left\{ \frac{\cos(\phi) \sin(\theta)}{\cos((\pi/2) + \phi + \theta) \sin((\pi/2) + \theta - P)} + \frac{\cos(\phi) \sin((\pi/2) + \theta) \sin((\pi/2) + \theta - P + \Omega + 2\phi)}{\sin((\pi/2) + \theta - P) \sin((\pi/2) + \theta + \phi) \sin((\pi/2) + \theta - P + 2\phi)} \right\}.
\]  

(15)

The dependence of the upper bound of the passive rock pressure coefficient on the angles \(P, \Omega\) is graphically displayed in Figs. 23a,b for the two cases of internal friction angles. These figures clearly indicate the existence of minimum upper bounds for the two cases. The comparison of the passive rock pressure coefficients predicted by the numerical solution and the above analytical solution are displayed in Table 4. From this table it may be seen that both numerical and analytical approaches give results that are in very good agreement.
6. Concluding remarks

In this paper, data pertaining to small-diameter rotary drilling experiments performed by the new portable device DFTMS on four types of marbles are presented. Subsequently, both a plane continuum numerical FLAC model and a discrete element PFC model were elaborated that simulate rotary drill cutting of a continuous and a granular medium, respectively. Finally, an analytical model was elaborated based on the results of the continuum numerical model. The main purpose of this study is to examine how will the rock-cutting experimental results can be reproduced by numerical or analytical model results that are using as input the mechanical properties of rocks found from standard rock mechanics tests (forward problem). Provided that this objective is successful then one may say that a step forward has been made for using the portable rotary drilling tool for characterization of basic rock
mechanical parameters either in the lab or most importantly in situ. Needless to say that the portable microdrilling tool provides a continuous log of forces along the drillhole that may extend up-to 15 cm in the rock, hence giving not only the heterogeneity of the rock, but also an indicator of its strength profile along the hole. On the other hand, the standard uniaxial or triaxial compression tests give only a mean value of the strength of the rock representative of the whole specimen. The following main conclusions may be drawn from the present experimental and numerical study of cutting of hard crystalline marbles by rotary drilling, namely:

1. It was experimentally verified that for cutting depths per revolution \( \delta \) in the range 0.03–0.55 mm/rev, the normal and tangential components of the cutting force depend linearly on the cutting depth per revolution.

2. Experimental evidence also indicates (a) a linear constraint between the two cutting forces that is almost the same for the four types of marble, and (b) a nearly perfectly smooth contact between the bit and the rock.

3. The predictions of the 2D continuum model, that is essentially a linear elastic–perfectly plastic model with a linear Mohr–Coulomb failure criterion and a non-associative flow rule, are in full accordance with measured forces during drilling.

4. The most important parameters that influence the drilling resistance of rocks are: (a) the drilling operational parameters, i.e. the drill-bit radius \( a \), the rake angle of the drill bit \( \theta \) and the cutting depth per revolution \( \delta \), as well as (b) the basic rock strength properties such as the uniaxial compressive strength and the internal friction angle, which may be easily measured from triaxial compressive tests on rock specimens. More specifically, from the numerical simulations as well as from the Limit Analysis Upper Bound Theorem, it was found that the expressions for cutting forces depend only on the rock strength properties, drill-bit radius and cutting depth per revolution as follows:

\[
F_n = ca \delta K_{pc}(\phi) \sin(\theta),
\]
\[
F_s = ca \delta K_{pc}(\phi) \cos(\theta),
\]

where \( K_{pc}(\phi) \) is a function that depends on the internal friction angle \( \phi \) of the rock and is obtained from a best-fitting procedure. In order to check the validity of the above expressions, a comparison was performed in Fig. 24, of the predicted and measured forces on Pendelikon marble that is characterized by a cohesion \( c = 26 \text{ MPa} \) and an angle of internal friction of \( \phi = 28.8^\circ \). This comparison demonstrates the validity of analytical Eq. (16).

5. The kinematical failure mechanism predicted by the numerical model indicates that the rock fails under a continuous plastic crushing process rather than in a discontinuous brittle chipping process. Severe particle crushing during drilling is experimentally demonstrated by comparing the original grain size cumulative distribution with the particle size cumulative distribution of Gioia marble in Fig. 25. The latter is constructed by image analysis of the collected dust after a drilling test with \( \omega = 600 \text{ rpm} \), \( \nu = 10 \text{ mm/min} \) or \( \delta = 104.7 \mu \text{m} \), and the former by image analysis of a thin section. From this analysis it is inferred that the original mean grain size of marble is reduced approximately 30 times by rotary drill cutting.
6. The cutting forces predicted by the distinct element simulations are given with respect to the dimensionless ratio of cutting depth per revolution divided by the particle radius. Therefore, the best fit of numerical predictions on experimental data gives the mean particle radius of the model for each type of marble. The calibrated mean particle size in conjunction with the tensile strength of rock may be used for the prediction of the mode-I fracture toughness and specific surface energy for fracture for each marble. In two cases, the inferred fracture toughness is found to be close with the fracture toughness estimated from standard fracture mechanics tests.

Fig. A1. Calibration of the set of micromechanical parameters on the experimental tests for the three types of marbles.
7. All the two sets of predictions, namely those performed independently by FLAC simulating cutting as a steady-state process and PFC simulating cutting as a destructive (fracture mechanical) process are in good agreement with the experimental data.

More work remains in order to validate the above conclusions for the case of low-to-medium and high strength rocks of various lithologies (e.g. limestones, sandstones, granites, etc.) and to investigate the effects of tool wear on the experimental and theoretical results, as well as its modeling with the continuum and discrete numerical tools. It is remarked here that this investigation is very important in order the portable rotary microdrilling tool to become a standard tool for rock strength characterization in rock-engineering practice. This study indicates that at least for homogeneous rocks, such as marbles, this tool can be employed for the estimation of their cohesion, internal friction angle and fracture toughness.

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Appendix A

The calibrated discrete element models on the experimental uniaxial stress-strain curves in the loading regime of the remaining three marble types are illustrated in Figs. A1(a–j). As was done in the case of Dionysos marble the set of micromechanical parameters listed in Table 2 were estimated by best fitting the PFC2D models on:

1. the axial stress–axial strain curve in UC test,
2. the lateral strain–axial strain in UC test, and
3. the set of principal stresses at failure in triaxial compression tests.

References


